Transport theory of superconductors with singular interaction corrections

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We study effects of strong fluctuations on the transport properties of superconductors near the classical critical point. In this regime conductivity is set by the delicate interplay of two competing effects. The first is that strong electron-electron interactions in the Cooper channel increase the lifetime of fluctuation Cooper pairs and thus enhance conductivity. On the other hand, quantum pair-breaking effects tend to suppress superconductivity. An interplay between these processes defines new regime, $Gi \lesssim \frac{T-T_c}{T_c} \lesssim \sqrt{Gi}$, where fluctuation induced transport becomes more singular, here Gi is the Ginzburg number. The most singular contributions to the conductivity stem from the dynamic Aslamazov-Larkin term, and novel Maki-Thompson and interference corrections. The crossover temperature $T_c\sqrt{Gi}$ from weakly to strongly fluctuating regime is generated self-consistently as the result of scattering on dynamic variations of the order parameter. We suggest that the way to probe nonlinear-fluctuations in superconductors is by magnetoconductivity measurements in the perpendicular field.

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Introduction. In the context of transport properties of disordered fluctuating superconductors one usually discusses three types of contributions to the normal-state Drude conductivity $\sigma_D = e^2\nu D$ near T_c, see Ref. 1, here D is the diffusion coefficient and ν is the single-particle density of states. The first one, $\delta\sigma_{AL}$, is called Aslamazov-Larkin (AL) contribution.[2] It has a simple physical interpretation as the direct charge transfer mediated by the fluctuation Cooper pairs. Within the microscopic formulation, see corresponding diagram in the Fig. 1a, it reads analytically

$$\frac{\delta\sigma_{AL}}{\sigma_D} = \frac{\pi}{32\nu T_c^2}\sum_q Dq^2 \int \frac{d\omega}{\sinh^2\frac{\omega}{2T}}\left[\mathrm{Im}L^R_{q,\omega}\right]^2, \qquad (1)$$

where $L^R_{q,\omega} = -\frac{8T}{\pi}[Dq^2 + \tau_{\mathrm{GL}}^{-1} - i\omega]^{-1}$ is the retarded component of the interaction (fluctuation) propagator, and $\tau_{\mathrm{GL}}^{-1} = \frac{8T}{\pi}\ln\frac{T}{T_c} \backsimeq \frac{8}{\pi}(T - T_c)$ is inverse Ginzburg-Landau time. After the energy and momentum integrations Eq. (1) reduces to the celebrated result[2]

$$\frac{\delta\sigma_{AL}}{\sigma_D} = \frac{1}{2\pi g}(T_c\tau_{\mathrm{GL}}) = Gi\left(\frac{T_c}{T-T_c}\right), \qquad (2)$$

for the thin-film superconductors with the thickness $b \lesssim \sqrt{D\tau_{\mathrm{GL}}}$, where $g = 1/\nu D b$ is the dimensionless conductance and $Gi = 1/16g$ is the Ginzburg number. It is worth recalling that AL term can be calculated from the time-dependent Ginzburg-Landau theory and to some extent is classical.

The other two contributions have purely quantum origin. The Maki-Thompson (MT) correction to conductivity,[3] $\delta\sigma_{MT}$, can be understood as the coherent Andreev reflection of electrons on the local fluctuations of the order parameter. Its most singular part near T_c is given by

$$\frac{\delta\sigma_{MT}}{\sigma_D} = -\frac{1}{2\pi\nu T}\sum_q \int \frac{d\epsilon d\omega \coth\frac{\omega}{2T}}{\cosh^2\frac{\epsilon}{2T}}\left[\mathrm{Im}L^R_{q,\omega}\right]\left|C^R_{q,2\epsilon+\omega}\right|^2, \qquad (3)$$

which is shown diagrammatically in the Fig. 1b. Here $C^R_{q,\epsilon} = [Dq^2 + \tau_s^{-1} - i\epsilon]^{-1}$ is the Cooperon, which accounts for the scattering by impurities in the particle-particle channel [sum of the ladder-type diagrams in the Fig. 1d], and τ_s is the spin-flip time. After the integrations in 2d-case Eq. (3) reduces to[3]

$$\frac{\delta\sigma_{MT}}{\sigma_D} = \frac{1}{\pi g}\frac{T_c\tau_{\mathrm{GL}}}{1-\tau_{\mathrm{GL}}/\tau_s}\ln\left(\frac{\tau_s}{\tau_{\mathrm{GL}}}\right), \qquad (4)$$

which unlike AL term [Eq. (2)] exhibits strong sensitivity to the dephasing time and is formally divergent without pair-breaking processes (no magnetic impurities for example). This is famous feature of the MT correction.

Finally, the density of states (DOS) effects[4] originate from the depletion of the energy states near the Fermi level by superconductive fluctuations. It leads to the cor-

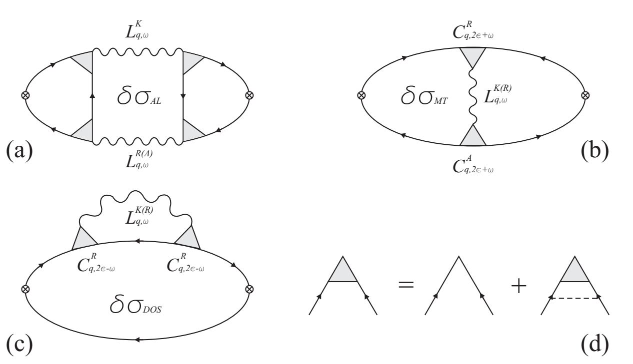

FIG. 1: Superconductive fluctuation corrections to the normal-state Drude conductivity: (a) Aslamazov-Larkin diagram, (b) Maki-Thompson correction, (c) density of states contribution, and (d) Cooperon impurity vertex.

rection to conductivity of the form [see Fig. 1c]

$$\frac{\delta\sigma_{DOS}}{\sigma_D}=\frac{1}{2\pi\nu T}\sum_q\int\frac{d\epsilon d\omega\coth\frac{\omega}{2T}}{\cosh^2\frac{\epsilon}{2T}}\big[\mathrm{Im}L^R_{q,\omega}\big]\mathrm{Re}\big(C^R_{q,2\epsilon-\omega}\big)^2\,, \tag{5}$$

which in contrast to AL and MT contributions is negative but has much weaker (logarithmic instead of the power-law) temperature dependence

$$\frac{\delta\sigma_{DOS}}{\sigma_D}=-\frac{7\zeta(3)}{\pi^4 g}\ln(T_c\tau_{\mathrm{GL}})\,, \tag{6}$$

where $\zeta(x)$ is the Riemann zeta function.

Applicability of the perturbative treatment for superconductive fluctuations implies that corresponding corrections to the conductivity [Eqs. (2), (4), and (6)] are small as compared to its bare Drude value. Thus, requirement that $\delta\sigma_{AL}+\delta\sigma_{MT}\lesssim\sigma_D$ restricts perturbation theory to the temperatures above the Ginzburg region, $T_cGi\lesssim T-T_c$. However, as it has been shown by Larkin and Ovchinnikov,[5] this conclusion is premature. It turns out that Eq. (2) is applicable only as long as $T-T_c\gtrsim T_c\sqrt{Gi}$ while $\delta\sigma_{AL}$ becomes more singular in the immediate vicinity of the critical temperature $Gi\lesssim\frac{T-T_c}{T_c}\lesssim\sqrt{Gi}$ where[5]

$$\frac{\delta\sigma_{AL}}{\sigma_D}\simeq\frac{1}{\pi^3g^2}(T_c\tau_{\mathrm{GL}})^2(T_c\tau_\phi),\quad \tau_\phi^{-1}=\max\{\tau_s^{-1},T\sqrt{Gi}\}\,. \tag{7}$$

In addition it was demonstrated by Reizer[6] that MT term saturates near T_c

$$\frac{\delta\sigma_{MT}}{\sigma_D}\simeq\frac{1}{2\pi\sqrt{g}}\ln\left(\frac{T_c\tau_{\mathrm{GL}}}{\sqrt{g}}\right),\quad Gi\lesssim\frac{T-T_c}{T_c}\lesssim\sqrt{Gi}, \tag{8}$$

even without an extrinsic pair-breaking, such as magnetic impurities. Thus, it was concluded that interaction corrections to the conductivity of fluctuating superconductors are governed by $\delta\sigma_{AL}$ [Eq. (7)] at $Gi\lesssim\frac{T-T_c}{T_c}\lesssim\sqrt{Gi}$. Stronger singularity of the AL term was attributed to the life-time enhancement of the preformed Cooper pairs by nonlinear-fluctuation effects.[5] At the same time, saturation of the interference MT contribution emerges as the result of scattering on dynamical variations of the order parameter, which generate an intrinsic dephasing time $\tau_\phi^{-1}\simeq T\sqrt{Gi}\simeq T/\sqrt{g}$.[6,7] In what follows we show that this physical picture of fluctuations-enhanced transport is incomplete. We identify novel class of interaction corrections, not discussed in the literature before, which strongly influence conductivity at the onset of superconducting transition. At the technical level diagrammatic analysis for the conductivity corrections is carried within nonlinear sigma model of fluctuating superconductors, see Ref. 8 for the review.

Singular interaction corrections. Apart from the conventional contributions to the conductivity, which appear to the first order in superconductive fluctuations, there are certain next leading order terms which make conductivity to be more singular near the transition. We find that among those the contributions shown in the Fig. 2 are the most important. Obviously, these terms carry and extra small pre-factor, $Gi\ll 1$, due to the perturbative treatment of fluctuation effects, however, they exhibit much stronger temperature dependence then $\delta\sigma_{AL}$ and $\delta\sigma_{MT}$, and become more important in the temperature region $Gi\lesssim\frac{T-T_c}{T_c}\lesssim\sqrt{Gi}$.

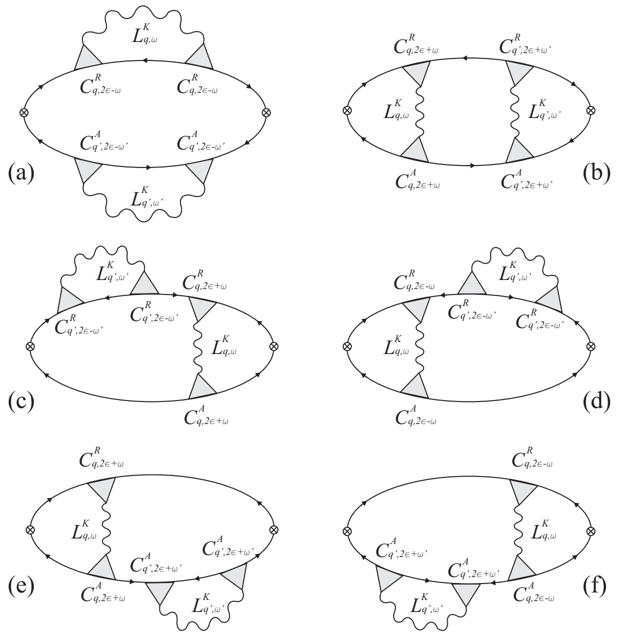

FIG. 2: Singular corrections to the fluctuation conductivity: diagrams (a) and (b) represent Maki-Thompson contributions due to interaction of fluctuations and their interference correspondingly; diagrams (c)-(f) represent mixture of the density of states and Maki-Thompson scattering processes.

We find two new MT contributions which depend differently on the dephasing time. The first one is presented in the Fig. 2a and its most singular part near T_c reads analytically as

$$\frac{\delta\sigma^a_{MT}}{\sigma_D}=-\frac{\pi}{\nu^2}\sum_{qq'}\int\frac{d\omega d\omega' d\epsilon}{\cosh^2\frac{\epsilon}{2T}}L^K_{q,\omega}L^K_{q',\omega'}\times(C^R_{q,2\epsilon-\omega})^2(C^A_{q',2\epsilon-\omega'})^2\,. \tag{9}$$

This term represents interaction of superconductive fluctuations. Notice here, that although diagrammatically $\delta\sigma^a_{MT}$ looks like second-order DOS effect, in fact, it should be classified as MT term by the analytical properties. Indeed, MT contributions involve mixture of retarded and advanced Cooperons while DOS terms always bring Cooperons of the same causality. This important difference makes DOS contributions to be subleading in powers of $T_c\tau_{\mathrm{GL}}$ [compare Eqs. (3) and (4) with Eqs. (5) and(6)]. It is worth recalling that $\delta\sigma^a_{MT}$ is already familiar from the studies of diffusive[9] and ballistic[10] tunnel

junctions, and granular superconductors[11] above T_c. Assuming static pair-breaking at this stage and after the consecutive energy integrations Eq. (9) reduces to

$$\frac{\delta\sigma^a_{MT}}{\sigma_D}=\frac{16T_c^3}{\pi\nu^2}\sum_{qq'}\frac{1}{(Dq^2+\tau_{\mathrm{GL}}^{-1})(Dq'^2+\tau_{\mathrm{GL}}^{-1})}\times\frac{1}{(DQ^2+\max\{\tau_s^{-1},\tau_{\mathrm{GL}}^{-1}\})^3}\,,\qquad(10)$$

where $Q^2=q^2+q'^2$. The remaining q sums depend significantly on the effective dimensionality. For the quasi-two-dimensional case we find

$$\frac{\delta\sigma^a_{MT}}{\sigma_D}=\frac{1}{\pi^3g^2}\begin{cases}\frac{\pi^2-9}{6}(T_c\tau_{\mathrm{GL}})^3 & \tau_s\gg\tau_{\mathrm{GL}},\\ (T_c\tau_s)^3\ln^2(\tau_{\mathrm{GL}}/\tau_s) & \tau_s\ll\tau_{\mathrm{GL}}\,.\end{cases}\qquad(11)$$

One special feature of this result is that it remains finite even in the absence of extrinsic phase breaking, when $\tau_s\to\infty$. This is unlike the other MT contribution shown in Fig. 2b, which represents an interference of superconductive fluctuations and its most singular part is given by

$$\frac{\delta\sigma^b_{MT}}{\sigma_D}=-\frac{\pi}{\nu^2}\sum_{qq'}\int\frac{d\omega d\omega' d\epsilon}{\cosh^2\frac{\epsilon+\omega'}{2T}}L^K_{q,\omega}L^K_{q',\omega'}\times|C^R_{q,2\epsilon+\omega}|^2|C^A_{q',2\epsilon+\omega'}|^2.\qquad(12)$$

Technically, it is different from Eq. (9) by the structure of the Cooperon propagators. Notice that the absolute value of $C^{R(A)}$ makes the integrand of Eq. (12) to be extended in the energy space whereas corresponding expression in Eq. (9) is short ranged due to the pole structure of the $(C^R)^2(C^A)^2$ product. This feature translates into the different temperature dependence of $\delta\sigma^b_{MT}$ than that of $\delta\sigma^a_{MT}$. Indeed, after energy integrations Eq. (12) can be brought to the form

$$\frac{\delta\sigma^b_{MT}}{\sigma_D}=\frac{16T_c^3}{\pi\nu^2}\sum_{qq'}\frac{1}{(Dq^2+\tau_{\mathrm{GL}}^{-1})(Dq'^2+\tau_{\mathrm{GL}}^{-1})}\frac{1}{(Dq^2+\tau_s^{-1})(Dq'^2+\tau_s^{-1})(DQ^2+\max\{\tau_s^{-1},\tau_{\mathrm{GL}}^{-1}\})}\,,\qquad(13)$$

which gives eventually in two dimensions with the logarithmic accuracy

$$\frac{\delta\sigma^b_{MT}}{\sigma_D}\simeq\frac{1}{\pi^3g^2}\begin{cases}(T_c\tau_{\mathrm{GL}})^3\ln^2(\tau_s/\tau_{\mathrm{GL}}) & \tau_s\gg\tau_{\mathrm{GL}},\\ (T_c\tau_s)^3\ln^2(\tau_{\mathrm{GL}}/\tau_s) & \tau_s\ll\tau_{\mathrm{GL}}\,.\end{cases}\qquad(14)$$

Clearly, when compared to the corresponding limit of Eq. (11), the divergence of $\delta\sigma^b_{MT}$ at weak pair-breaking $\tau_s\to\infty$, is the manifestation of coherence.

The remaining four terms in Figs. 2c-2f represent the mixture of MT and DOS contributions, we thus label those collectively by $\delta\sigma_{MTD}$. Although each individual diagram has slightly different analytical structure, their most divergent parts, however, are the same for all four terms. We find then for the sum of these contributions, $\delta\sigma_{MTD}\simeq 4\delta\sigma_{\mathrm{Fig-2c}}$, following expression

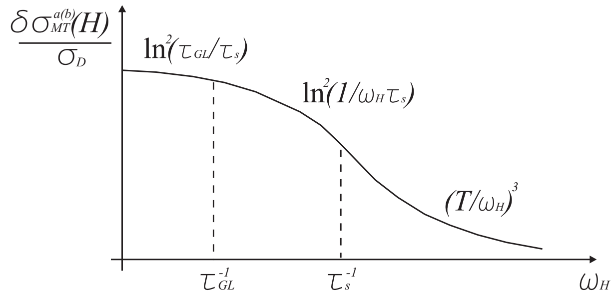

FIG. 3: Sketch for the characteristic magnetic field dependence of the singular Maki-Thompson correction to the conductivity [Eqs. (18)] under the condition of strong phase-breaking scattering $\tau_{\mathrm{GL}}\gg\tau_s$. In the opposite case one should replace $\tau_{\mathrm{GL}}\rightleftarrows\tau_s$.

$$\frac{\delta\sigma_{MTD}}{\sigma_D}=\frac{4\pi}{3\nu^2}\sum_{qq'}\int\frac{d\omega d\omega' d\epsilon}{\cosh^2\frac{\epsilon+\omega}{2T}}L^K_{q,\omega}L^K_{q',\omega'}\times|C^R_{q,2\epsilon+\omega}|^2(C^R_{q',2\epsilon-\omega'})^2\,.\qquad(15)$$

After the standard steps of integration this equation reduces to

$$\frac{\delta\sigma_{MTD}}{\sigma_D}=-\frac{32T_c^3}{3\pi\nu^2}\sum_{qq'}\frac{1}{(Dq^2+\tau_{\mathrm{GL}}^{-1})(Dq'^2+\tau_{\mathrm{GL}}^{-1})}\frac{1}{(Dq^2+\tau_s^{-1})(DQ^2+\max\{\tau_s^{-1},\tau_{\mathrm{GL}}^{-1}\})^2}\,,\qquad(16)$$

which gives for the conductivity correction of a thin superconducting film

$$\frac{\delta\sigma_{MTD}}{\sigma_D}\simeq-\frac{1}{3\pi^3g^2}\begin{cases}(T_c\tau_{\mathrm{GL}})^3\ln(\tau_s/\tau_{\mathrm{GL}}) & \tau_s\gg\tau_{\mathrm{GL}},\\ 2(T_c\tau_s)^3\ln^2(\tau_{\mathrm{GL}}/\tau_s) & \tau_s\ll\tau_{\mathrm{GL}}\,.\end{cases}\qquad(17)$$

We see from here that mixed contributions suppress fluctuation conductivity, unlike MT terms. Second, mixed terms exhibit weaker divergence then $\delta\sigma^b_{MT}$ for the small static pair-breaking $\tau_s\to\infty$. These singular MT and mixed contributions [Eqs. (11), (14), and (17)] together with the AL correction Eq. (7) represent the leading terms in the asymptotic expansion of fluctuation conductivity at the onset of superconducting transition.[12]

Magnetoconductivity. Since $\delta\sigma^{a(b)}_{MT}$ and $\delta\sigma_{MTD}$ have distinct temperature dependence there is a way to identify these terms by the appropriate transport experiment. The most suitable one would be the magnetoconductivity measurement in the perpendicular field. Indeed, magnetic field acts as an effective pair-breaking mechanism which drives a superconductor away from the critical region. This is simply understood by looking at the pole structure of the interaction propagator

$L^{R(A)}(q,\omega)\propto(Dq^2+\tau_{\rm GL}^{-1}\mp i\omega)^{-1}$ and recalling that magnetic field H applied perpendicularly to a film changes the continuous spectrum of superconducting fluctuations into its quantized form $Dq^2\to\omega_n=\omega_H(n+1/2)$, where $\omega_H=4eDH$ is the cyclotron frequency and $n=0,1,2,\ldots$ is the number of the Landau level. It becomes clear now that if cyclotron frequency ω_H exceeds $\tau_{\rm GL}^{-1}$, it is ω_H that cuts all energy transfer integrations, since $\omega\sim\max\{\omega_H,\tau_{\rm GL}^{-1}\}$. Roughly speaking it means that in the expressions for the conductivity corrections Eqs. (11), (14) and (17) the scale of $T-T_c$ is replaced by ω_H. This gives a possibility to restore the temperature dependence of $\delta\sigma(T)$ by observing its behavior as the function of magnetic H, which traces corresponding dependence on $T-T_c$. In what follows we calculate $\delta\sigma^a_{MT}(H)$ explicitly and quote only final results for the remaining contributions.

Starting from Eq. (10) we replace momentum integration by the discrete sum over the Landau levels $\sum_q\to\frac{\omega_H}{4\pi Db}\sum_{n=0}^{\infty}$, where the prefactor accounts for the degeneracy in the position of the orbit, and find

$$\frac{\delta\sigma^a_{MT}(H)}{\sigma_D}=\frac{\omega_H^2T_c^3}{\pi^3g^2}\sum_{nn'=0}^{\infty}\frac{1}{(\omega_n+\tau_{\rm GL}^{-1})(\omega_{n'}+\tau_{\rm GL}^{-1})}\frac{1}{(\Omega_{nn'}+\max\{\tau_s^{-1},\tau_{\rm GL}^{-1}\})^3},\quad(18)$$

where $\Omega_{nn'}=\omega_n+\omega_{n'}$. If $\omega_H\ll\max\{\tau_{\rm GL}^{-1},\tau_s^{-1}\}$, which corresponds to the zero-field limit, we restore Eq. (11). At higher fields there are two regimes. For $\tau_{\rm GL}^{-1}\ll\omega_H\ll\tau_s^{-1}$ quantization of the spectrum of fluctuations is already important in the interaction propagator while vertex Cooperons can still be taken at zero field. As the result $\delta\sigma^a_{MT}(H)$ turns out to be logarithmic in H. At even higher fields $\omega_H\gg\tau_s^{-1}$ superconducting fluctuations are strongly suppressed and corresponding correction decays inversely proportional to the third power of magnetic field. Quantitatively we find following asymptotes for these limits:

$$\frac{\delta\sigma^a_{MT}(H)}{\sigma_D}=\frac{1}{\pi^3g^2}\begin{cases}(T_c\tau_s)^3\ln^2\frac{1}{\omega_H\tau_s} & \tau_{\rm GL}^{-1}\ll\omega_H\ll\tau_s^{-1}\,,\\ 4.46(T_c/\omega_H)^3 & \omega_H\gg\tau_s^{-1}\,.\end{cases}\quad(19)$$

Similar analysis can be carried for Eqs. (13) and (16). We find that interference contribution $\delta\sigma^b_{MT}(H)$ follows the same asymptotes as $\delta\sigma^a_{MT}(H)$ while mixed term $\delta\sigma_{MTD}(H)$ is negative and differs from Eq. (19) only by the numerical coefficient $2/3$ in the limit $\tau_{\rm GL}^{-1}\ll\omega_H\ll\tau_s^{-1}$, and 1.4 in the limit $\omega_H\gg\tau_s^{-1}$. The magnetic field dependence of the conventional AL and MT contributions was recently discussed in Ref. 14. Finally, Fig. 3 schematically shows anomalous correction to the conductivity induced by the interacting fluctuations in the whole range of magnetic fields.

Regularization. Without an extrinsic static pair-breaking $\tau_s\to\infty$ physical origin of the divergent MT contribution $\delta\sigma^b_{MT}$ and mixed terms $\delta\sigma_{MTD}$ comes from the softness of the Cooperon. This is exactly the same problem that exists for the conventional MT contribution Eq. (4) and thus, regularization is achieved by following the prescription given in the Ref. 6, which allowed to regularize $\delta\sigma_{MT}$ [Eq. (8)]. The main idea is to include Cooperon self-energy[6,7,15]

$$\Sigma_{\epsilon,-\epsilon}=\frac{1}{\pi\nu}\sum_q\int d\omega\left[\coth\frac{\omega}{2T}+\tanh\frac{\epsilon-\omega}{2T}\right]\times{\rm Im}[L^A_{q,\omega}]{\rm Re}[C^R_{q,2\epsilon-\omega}]\quad(20)$$

into the general scheme of calculations. It is important to realize that this object is strongly ϵ dependent in the broad range of energies[6]

$$\Sigma_{\epsilon,-\epsilon}=\frac{T^2}{2\pi g|\epsilon|}\,,\quad\tau_{\rm GL}^{-1}\lesssim\epsilon\ll T\,.\quad(21)$$

Since $\Sigma_{\epsilon,-\epsilon}$ enters now the Cooperon instead of τ_s^{-1} an integration in Eq. (12) over the fermionic energy ϵ must be completed carefully. An inspection of the integrand reveals that $\epsilon\simeq T/\sqrt{g}$ give the most important contribution. After an explicit calculation we find

$$\frac{\delta\sigma^b_{MT}}{\sigma_D}\simeq\frac{1}{4\sqrt{2}\pi^{3/2}}\frac{1}{\sqrt{g}}\ln^2\left(\frac{\tau_{\rm GL}}{\tau_\phi}\right),\quad\tau_\phi^{-1}\simeq\frac{T}{\sqrt{g}}\,,\quad(22)$$

which is applicable in the temperature range $Gi\lesssim\frac{T-T_c}{T_c}\lesssim\sqrt{Gi}$. Thus, inclusion of $\Sigma_{\epsilon,-\epsilon}$ self-consistently generates an intrinsic dephasing time τ_ϕ which regularizes $\delta\sigma^b_{MT}$. Even more importantly, quantum dynamic pair-breaking encoded by the Cooperon self-energy changes dramatically the temperature dependence of this singular interaction correction and leads to its saturation apparent from Eq. (22) [$\delta\sigma^a_{MT}$ also saturates in this case as shown in Ref. 6]. The same regularization approach applied to the mixed term $\delta\sigma_{MTD}$ gives a contribution that is three times smaller then $\delta\sigma_{MTD}$, which concludes our analysis.

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[12] It is rather interesting that MT contributions remain to be important even far away from T_c and even in the case of repulsion between electrons. Larkin [Ref. 13] had shown that conventional MT term [Eq. 3] partially compensates for the contribution to conductivity due to weak-localization. This observation significantly improved an agreement between the theory and experiment. Due to its potential importance and for completeness we also calculated $\delta\sigma^{a(b)}_{MT}$ away from the critical region, $\ln\frac{T}{T_c}\gg 1$, and found:

$$\frac{\delta\sigma^{a(b)}_{MT}}{\sigma_D}\backsimeq\frac{c_{a(b)}}{g^2}\frac{\ln^2[\lambda T\tau_s]}{\ln^4(T/T_c)}\,,\quad \lambda=\min\{4\pi,\ln(T/T_c)\}\,,$$

with coefficients $c_a=(13\pi^2+12)/36864$ and $c_b=(\pi^2+12)/9216$.

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